Global-Local Approach to Solving Vibration of Large Truss Structures

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A global-local approach is proposed to solve dynamic problems involving truss beams. A continuum Timoshenko beam used to model sections of truss beams wherever possible. (The continuum model was not adequate near applied loads, and detailed truss finite elements [discrete model] were employed.) This approach was also proven efficient for an odd-shaped truss structure connected to a truss beam.

I. Introduction

MANY large truss structures have the function of a beam or plate. It has been common to use the finite-element method to perform static as well as dynamic analyses on these structures in which each truss member is modeled by an axial element. If the number of truss members in the structure is large, such analysis can become computationally expensive. Moreover, in dynamic analysis, the finite-element method may produce more modes of vibration than are actually needed. The continuum approach, in which the actual truss structure is replaced with an equivalent continuous beam or plate, has been found suitable for analyzing large truss structures.

Several methods have been proposed to construct continuum models to represent discrete structures.[1-11] In one of the methods, a Timoshenko beam or plate model was assigned a priori for a beam-like or plate-like structure, and then the beam or plate properties were evaluated by studying the behavior of typical cells of the structure.[8] Although the continuum approach has proven to be adequate in describing global responses such as natural frequencies and mode shapes, it may not yield accurate results for loads in truss members in the neighborhood of applied loads. Also, if the truss beam is connected to a truss structure of an arbitrary shape, the application of continuum models for the entire structure may become less accurate.

In this study, attempts are made to examine the effect of continuum-discrete modeling on the accuracy of the model response and element forces in large truss structures. The proposed procedure employs a continuum beam model wherever the structure takes the configuration of a beam, and detailed discrete finite elements for other forms. Numerical examples are presented for evaluative purposes.

II. Extended Timoshenko Beam Model

The conventional Timoshenko beam theory was developed using homogenous isotropic materials in which the three basic deformations, i.e., extension, transverse shear, and bending, were not coupled. A more general beam theory is needed to model an arbitrary truss beam in which the three basic deformations may be coupled. The force-deformation relations that account for these coupling effects are given by

$$\begin{Bmatrix} N \\ Q \\ M \end{Bmatrix} = \begin{bmatrix} EA & \eta_{12} & \eta_{13} \\ \eta_{12} & GA & \eta_{23} \\ \eta_{13} & \eta_{23} & EI \end{bmatrix} \begin{Bmatrix} u_{,x} \\ w_{,x} + \psi \\ \psi_{,x} \end{Bmatrix} \tag{1}$$

where

EA = longitudinal rigidity
EI = bending rigidity
GA = transverse shear rigidity
M = bending moment
N = extensional force
Q = transverse shear force
u = longitudinal displacement
w = transverse displacement
Ψ = rotation of cross section
η_{ij} = coupling coefficients

In Eq. (1) a comma indicates partial differentiation.

The corresponding equations of motion are

$$N_{,x} + q_x = m\ddot{u} + R\ddot{\Psi} \tag{2a}$$

$$Q_{,x} + q_z = m\ddot{w} \tag{2b}$$

$$M_{,x} - Q = R\ddot{u} + \rho I\ddot{\Psi} \tag{2c}$$

in which

m = mass per unit length
q_x = externally applied force per unit length in the longitudinal direction
q_z = transverse load per unit length
ρI = mass moment of inertia of cross section

$$R = \int_{-h/2}^{h/2} \rho z \, dz$$

where h is the thickness of the beam.

Presented as AIAA Paper 86-0872 at the AIAA/ASME/ASCE/AHS 27th Structures, Structural Dynamics, and Materials Conference, San Antonio, TX, May 19–21, 1986; received June 22, 1987; revision received Aug. 15, 1988.

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Substituting Eq. (1) into Eqs. (2) yields the displacement equation of motion as

$$\begin{bmatrix} \partial_x(EA\,\partial_x) & \partial_x(\eta_{12}\partial_x) & \partial_x(\eta_{12}+\eta_{13}\partial_x) \\ \partial_x(\eta_{12}\partial_x) & \partial_x(GA\,\partial_x) & \partial_x(GA+\eta_{23}\partial_x) \\ -\eta_{12}\partial_x+\partial_x(\eta_{13}\partial_x) & -GA\,\partial_x+\partial_x(\partial_x(\eta_{23}\partial_x) & -GA+\partial_x(EI\partial_x) \end{bmatrix} \begin{Bmatrix} u \\ w \\ \psi \end{Bmatrix} = \begin{bmatrix} m & 0 & R \\ 0 & m & 0 \\ R & 0 & \rho I \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{w} \\ \ddot{\psi} \end{Bmatrix} - \begin{Bmatrix} q_x \\ q_z \\ 0 \end{Bmatrix} \tag{3}$$

where ∂_x is a partial differential operator with respect to x, and a dot indicates differentiation with respect to time.

For a beam-like truss structure possessing a plane of symmetry coinciding with its midplane, the coupling coefficients η_{ij} vanish, and the three basic beam deformations are uncoupled, leaving the three stiffness coefficients, EA, GA, and EI, to be determined. In this case, these stiffness coefficients can be obtained by isolating a typical substructure and studying its force–deformation behavior in each type of deformation.

If coupling coefficients η_{ij} are present, it is more convenient to use the inverse relation of Eq. (1), i.e.,

$$\begin{Bmatrix} u_{,x} \\ w_{,x}+\Psi \\ \Psi_{,x} \end{Bmatrix} = [\eta^*] \begin{Bmatrix} N \\ Q \\ M \end{Bmatrix} \tag{4}$$

where

$$[\eta^*] = [\eta]^{-1} \tag{5}$$

and $[\eta]$ is the 3×3 beam stiffness matrix in Eq. (1). The elements η^*_{ij} can be evaluated column by column by applying a unit load for N, Q, and M, respectively, to the typical substructure. For instance, by applying $N=1$ and $Q=M=0$, the resulting values of $u_{,x}$, $w_{,x}+\Psi$, and $\Psi_{,x}$ are equal in value to η^*_{11}, η^*_{21}, and η^*_{31}, respectively. The other η^*_{ij} can be obtained in a similar manner. The elements in $[\eta]$ are then obtained by inverting $[\eta^*]$. Note that if a truss member is shared by two adjacent substructures, the cross-sectional area of this member should be reduced by half.

In the above procedure, care must be exercised in interpreting the beam deformations $u_{,x}$, $w_{,x}+\Psi$, and $\Psi_{,x}$. Figure 1 depicts the deformations of a unit cell of a truss beam resulting from application of a shear force Q, moment M, and extensional force N. Consider the case shown in Fig. 1c. Because of a unit extensional force applied to the unit cell, the resulting extension is δu, and the rotation at the left end is $\delta\Psi_L$ and at the right end is $\delta\Psi_R$; the vertical displacements of the midplane at the left and right ends are δw_L and δw_R (not shown in the figure), respectively.

From this deformed geometry, we obtain

$$\eta^*_{11} = u_{,x} \approx \delta u / L_c \tag{6a}$$

$$\eta^*_{21} = w_{,x} + \Psi \approx (\delta w_R - \delta w_L)/L_c + (\delta\Psi_R + \delta\Psi_L)/2 \tag{6b}$$

$$\eta^*_{31} = \Psi_{,x} \approx (\delta\Psi_R - \delta\Psi_L)/L_c \tag{6c}$$

To obtain η^*_{21}, η^*_{22}, and η^*_{23}, a unit moment is applied as shown in Fig. 1b.

The application of the transverse shear force is more tricky. Because of the finite dimension of the substructure under consideration, the unit shear force applied at the right end produces a couple at the left end. Thus, a pair of forces of 0.5 L_c/L_g as shown in Fig. 1a needs to be added in order to produce a state of shear stress in a continuum. If $L_c \to 0$, then forces vanish as expected. This pair of horizontal forces correspond to thickness shear stress in continuum theory and should not be confused with the beam bending moment.

It is important to note that the effective stiffness EA, GA, and EI should be regarded as single entities rather than the product of two constants.

The mass inertia terms for the continuum beam, i.e., R, m, and ρI, are calculated from the typical substructure by calculating the total inertias of the whole substructure first and then distributing them uniformly along the beam element.

Solutions to the continuum beam model can be obtained from the partial differential equations given by Eq. (1). In this study, we employed a high-order-beam finite element developed according to the continuum model.[12]

III. Determining Truss Loads from Continuum Displacements

For structural design purposes, loads carried by truss members that have been replaced with continuum elements may be needed. The member load in the actual truss can be obtained from the continuum model solution.

The solution for a Timoshenko beam is given in terms of the longitudinal displacement $u(x)$, the transverse displacement $w(x)$, both at the midplane of the beam, and the rotation of the cross section, $\Psi(x)$. The displacement at any joint of the truss can be calculated from the following relations

$$u^*(x,z) = u(x) + z\Psi(x) \tag{7a}$$

$$w^*(x,z) = w(x) \tag{7b}$$

where the coordinates of the joint (x,z) must be specified.

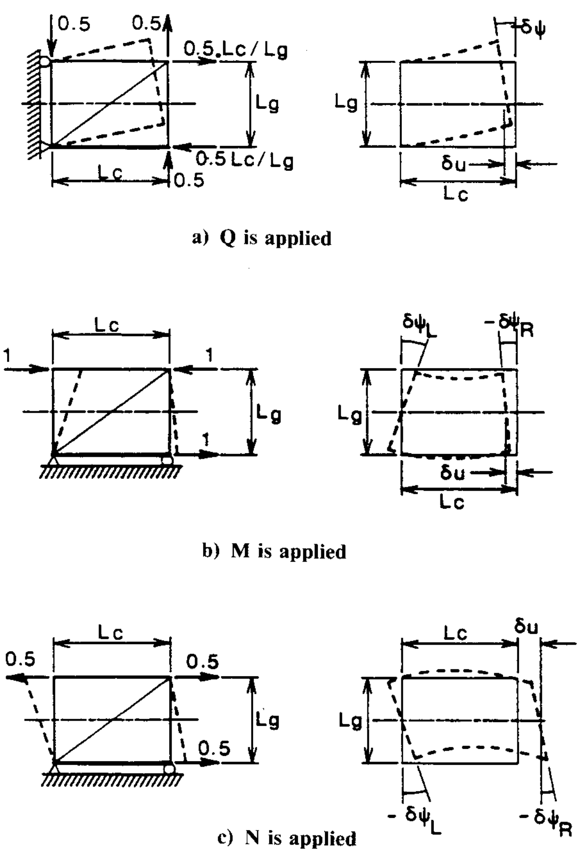


Fig. 1 Loads and boundary conditions applied to determine continuum properties.

For a truss bar joining the i-joint and the j-joint (see Fig. 2), the load in the bar is given by

$$S_{ij} = \frac{AE}{L}\left[(u_j^* - u_i^*)\cos\theta + (w_j^* - w_i^*)\sin\theta\right] \tag{8}$$

where A is the cross-sectional area of the bar, E the Young's modulus, and L the length. It is noted that the quantity AE in Eq. (8) is the true extensional stiffness of the truss bar, whereas the quantity EA in the continuum Timoshenko beam model is an equivalent extensional stiffness and should not be considered as the product of E and A.

IV. Global-Local Approach

In the global-local approach, a truss structure is divided into two or more sections. Each section is modeled using either continuum beam elements or conventional truss elements. Usually, sections that have external loads or sections that cannot be modeled by the continuum model are left as discrete elements. Sections that have many repeating cells and behave in a beam-like manner are replaced with continuum elements.

All sections that are modeled with continuum beam elements should act in a beam-like manner in their gross behavior. This means that the displacement field must follow the Timoshenko beam assumptions.

Consider the example of a truss structure shown in Fig. 3a. At both ends of the structure and where the load is applied, the boundary conditions and/or external forces will create localized deformations that violate the beam assumptions. Using the global-local approach, the sections on both ends of the structure and where the load is applied will be modeled using truss bar elements, and the rest will be modeled using continuum elements.

An idealization of how the structure will look after replacing appropriate sections with continuum beam elements is shown in Fig. 3b. The dashed portions are the continuum sections.

The number of continuum elements used to model the continuum sections depends on the complexity of the structure and the external loading. The continuum properties can be found using the methods outlined in Sec. II.

At the boundary where the continuum beam and truss bar elements join, a transformation of the truss nodal reactions and displacements must be done. Since all of the displacements and reactions are related along a continuum boundary due to the assumption of beam behavior, this is a relatively simple procedure.

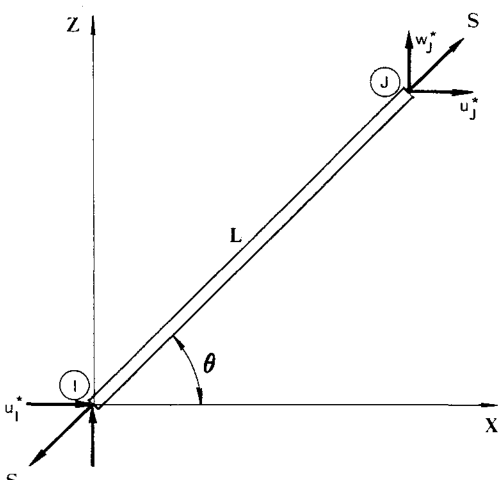
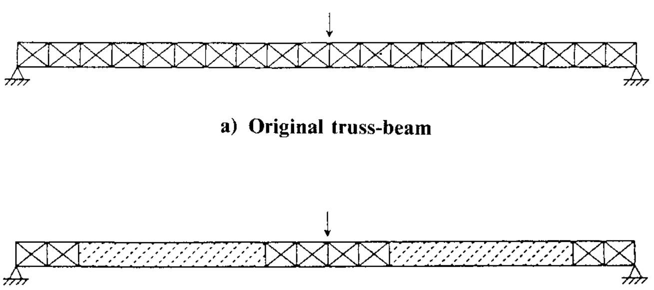


Fig. 2 Forces in an arbitrarily oriented truss element.



a) Original truss-beam

b) Idealized Global-Local truss beam

Fig. 3 Global-local representation of a truss structure.

V. Global-Local Boundary Node Displacements and Reactions

Consider a boundary where a continuum beam element and a truss element are connected as shown in Fig. 4. In Fig. 4, d_y is the distance between the continuum beam midplane and the truss node in the transverse direction. Note that the boundary between the two types of elements for this study is restricted to a vertical direction with respect to the global axis, although generalizing to an arbitrary direction is possible.

Any point that lies on the boundary between a truss bar and continuum beam element also lies on a line perpendicular to the midplane of the continuum beam element, and the line passes through one of the beam element nodes. Therefore, for any truss bar node that is on the boundary, its displacements can be expressed in terms of equivalent displacements and rotation of the continuum beam. Similarly, the reactions of the truss bar can also be expressed in terms of equivalent reactions located at the beam midplane.

Consider truss element 1–2 in Fig. 4 for which the element dynamic equilibrium equations are given by

$$\{F\} = [K]\{\Delta\} + [M]\{\ddot{\Delta}\} \tag{9}$$

$$\{F\} = \begin{Bmatrix} N_1 \\ Q_1 \\ N_2 \\ Q_2 \end{Bmatrix}, \qquad \{\Delta\} = \begin{Bmatrix} u_1 \\ w_1 \\ u_2 \\ w_2 \end{Bmatrix}, \qquad \{\ddot{\Delta}\} = \begin{Bmatrix} \ddot{u}_1 \\ \ddot{w}_1 \\ \ddot{u}_2 \\ \ddot{w}_2 \end{Bmatrix} \tag{10}$$

where $[K]$ is the element stiffness matrix, and $[M]$ is the element mass matrix of an arbitrarily oriented truss bar element. The matrices $[K]$ and $[M]$ can be derived using many methods. The expressions can be found in most finite element textbooks. Before assembling the element's stiffness and mass matrices, into the global stiffness and mass matrices, it is necessary to transform the displacements and reactions at truss node 1 to equivalent displacements and reactions about the continuum beam midplane. This results in a modified element stiffness and mass matrix that is then assembled with other element

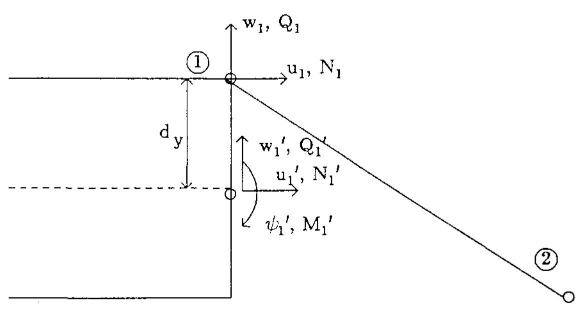


Fig. 4 Global-local boundary.

matrices to form the global stiffness and mass matrices. The effect of these transformations is to transform the boundary node's displacements and reactions into equivalent nodal displacements, forces, and moment located at the beam midplane. These transformations will remain valid as long as the structure continues to respond in a beam-like manner. This implies that the vertical distance between a truss node that lies on a global-local boundary to the midplane of the beam element remains constant after any deformation.

The relations used to perform the transformations are as follows. From Fig. 4, using simple geometry and the assumption of small displacements, it is easy to see that the equivalent displacements are

$$u_1 \approx u'_1 + d_y \Psi'_1, \qquad w_1 \approx w'_1 \tag{11}$$

$$N_1 = N'_1, \qquad Q_1 = Q'_1, \qquad d_y N_1 = M'_1 \tag{12}$$

where u'_1, w'_1, Ψ'_1, N'_1, Q'_1, and M'_1 are the equivalent displacements and resultants required to describe the nodal displacements and forces of the truss element if they were repositioned at the beam midplane. Hence, they are consistent with the Timoshenko beam displacements and forces.

To perform the transformation, the following force vector and displacement vector for truss element 1-2 is introduced.

$$\{F'\} = \begin{Bmatrix} N'_1 \\ Q'_1 \\ M'_1 \\ N_2 \\ Q_2 \end{Bmatrix}, \qquad \{\Delta'\} = \begin{Bmatrix} u'_1 \\ w'_1 \\ \Psi'_1 \\ u_2 \\ w_2 \end{Bmatrix}, \qquad \{\ddot{\Delta}'\} = \begin{Bmatrix} \ddot{u}'_1 \\ \ddot{w}'_1 \\ \ddot{\Psi}'_1 \\ \ddot{u}_2 \\ \ddot{w}_2 \end{Bmatrix} \tag{13}$$

Using the relations given by Eqs. (11) and (12), the force and displacement vectors in Eqs. (10) can be related to those of Eq. (13) by

$$\{F'\} = [T_F]\{F\}, \qquad \{\Delta\} = [T_\Delta]\{\Delta'\} \tag{14}$$

where

$$[T_F] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ d_y & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad [T_\Delta] = \begin{bmatrix} 1 & 0 & d_y & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

Substituting Eqs. (14) into Eq. (9), the element dynamic equilibrium equations can be expressed as

$$\{F'\} = [K']\{\Delta'\} + [M']\{\ddot{\Delta}'\} \tag{16}$$

where

$$[K'] = [T_F][K][T_\Delta], \qquad [M'] = [T_F][M][T_\Delta]$$

Similar procedures are followed on any other truss elements that contain nodes on the global-local boundary. After assemblage of the element matrices, the system of equations can be solved simultaneously for the displacements u'_1 and w'_1 and the rotation ψ'_1. Substituting these values into Eq. (11), the displacements and, thus, the forces of truss bar element 1-2 at node 1 can be obtained.

VI. Results

A. Symmetric Simply Supported Truss Beam

Consider a simply supported truss beam that is composed of 20 cells of symmetric truss elements as shown in Fig. 3a. Because of the symmetry of the truss beam, only half of the beam needs to be modeled, and appropriate boundary conditions applied. The truss beam dimensions and material properties as modeled are shown in Fig. 5. In the figure A_c, A_d, A_g are cross-sectional areas, E is the modulus of elasticity, and ρ is the mass density. The equivalent Timoshenko beam model properties are computed as explained previously. They are

$$EA = 1.477 \times 10^7 \text{ N}$$
$$EI = 7.170 \times 10^7 \text{ N-m}^2$$
$$GA = 1.469 \times 10^6 \text{ N}$$
$$\rho A = 0.820 \text{ kg/m}$$
$$\eta_{ij} = 0, \ \rho I = 3.553 \text{ kg-m}$$

Note that a slight error in ρA given in Refs. 8 and 11 is corrected here.

Three finite element models of the beam were used. The first used exclusively conventional truss finite elements, modeling each truss member by a single finite element. The second replaced the six center cells of the beam with six continuum elements, leaving two discrete cells as shown in Fig. 3b. The third replaced each cell in the structure with one continuum element. The discrete model is shown in Fig. 5.

The five natural frequencies corresponding to the five lowest modes of free vibration of the truss beam were determined. The results are shown in Table 1. All values for both models using the continuum elements are within 2% of those for the discrete model. It is seen that the global-local method gives values close to those of the continuum model.

A transient force was then applied in the center of the beam as shown in Fig. 3a. This force obeyed the forcing function

$$F(t) = 100.0 \sin(\pi/T_o)t \qquad t \le T_o$$
$$= 0.0 \qquad t > T_o \tag{17}$$

where T_o was chosen to be 0.5 s.

The axial forces produced in a few members of the truss are shown in Fig. 6. The element numbers referred to are shown in Fig. 5. As can be seen, the results for the two models containing continuum elements are in close agreement with those of the discrete model. Some discrepancies are observed in the diagonal elements of the cells where the external loads are applied when only continuum elements are used. It is likely that these errors are produced by the local non-beam-like behavior created by the external force which the continuum model cannot account for. In Figs. 6a and 6b, the combined global-local model is seen to be much more accurate.

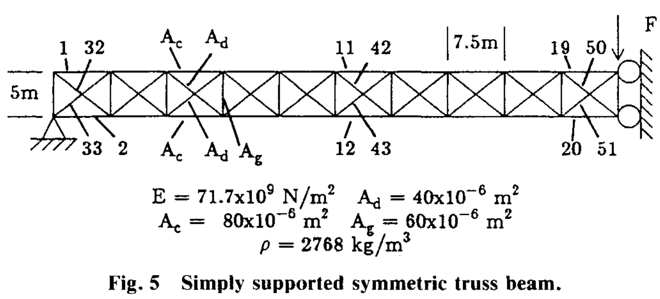


Fig. 5 Simply supported symmetric truss beam.

Table 1 Natural frequencies of symmetric simply supported truss beam (radians)

Mode	Discrete only	Global-local	Continuum only
1	4.03	4.06	4.06
2	33.5	33.6	33.6
3	81.5	81.9	81.9
4	86.5	87.3	87.1
5	137.0	138.0	139.0

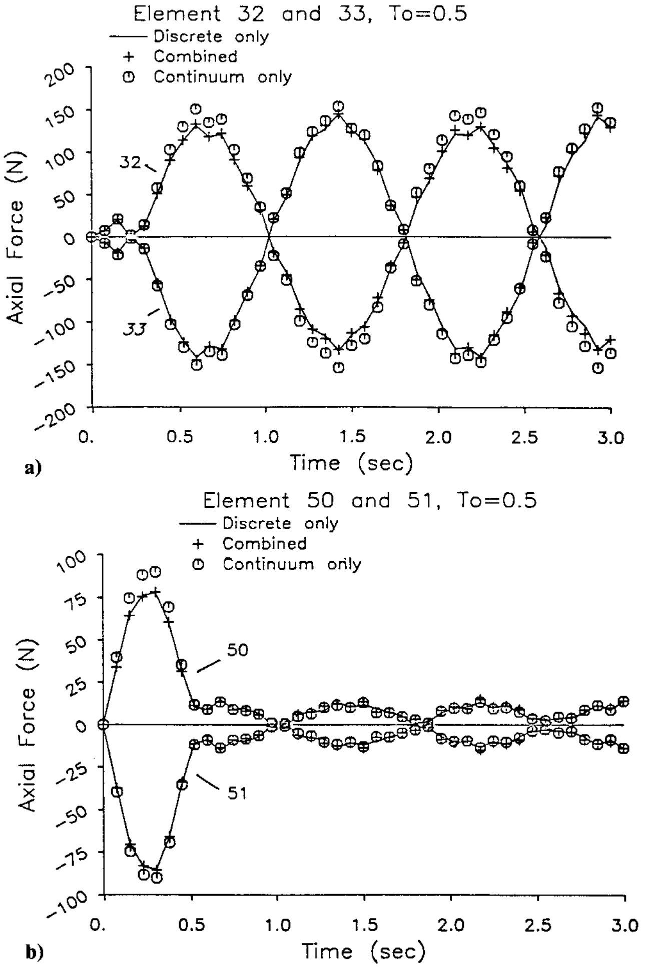


Fig. 6 Axial forces in elements of the simply supported symmetric truss beam.

B. Asymmetric Cantilever Truss Beam

In this example, a cantilever truss beam as shown in Fig. 7 was modeled, with material constants and geometry as shown. The equivalent continuum properties were found to be

$$EA = 2.013 \times 10^7 \text{ N}$$

$$EI = 11.651 \times 10^7 \text{ N-m}^2$$

$$GA = 6.592 \times 10^5 \text{ N}$$

$$m = 0.963 \text{ kg/m}, \; R = -0.692 \text{ kg}$$

$$\eta_{12} = 9.889 \times 10^5 \text{ N}$$

$$\eta_{13} = -1.792 \times 10^7 \text{ N-m}$$

$$\eta_{23} = 8.851 \text{ N-m}, \; \rho I = 5.008 \text{ kg-m}$$

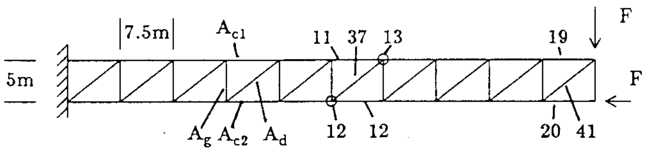
Once again, three models were used. The first was a discrete-element model, each truss element being modeled by a single truss finite element. The second was a combined global-local model, with all but the two cells closest to the free end modeled with continuum elements. The third used only continuum elements to model the truss beam. One continuum element was used to model each of the cells.

The five natural frequencies corresponding to the five lowest modes of free vibration of the truss beam were determined. The results are shown in Table 2.

For transient response, a pair of forces was applied at the free end of the beam as shown in Fig. 7. The magnitude of the forces are given by Eq. (17). Note that in the continuum model, these forces were transformed to a force and moment applied to the single node at its free end.

Table 2 Natural frequencies of asymmetric cantilever truss beam (radians)

Mode	Discrete only	Global-local	Continuum only
1	5.67	5.72	5.75
2	27.6	27.5	27.9
3	61.0	60.8	62.2
4	92.1	92.7	93.0
5	95.4	96.1	98.3



$E = 71.7 \times 10^9 \text{ N/m}^2 \quad \rho = 2768 \text{ kg/m}^3$
$A_{c1} = 80 \times 10^{-6} \text{ m}^2 \quad A_d = 40 \times 10^{-6} \text{ m}^2$
$A_{c2} = 180 \times 10^{-6} \text{ m}^2 \quad A_g = 60 \times 10^{-6} \text{ m}^2$

Fig. 7 Cantilever asymmetric truss beam.

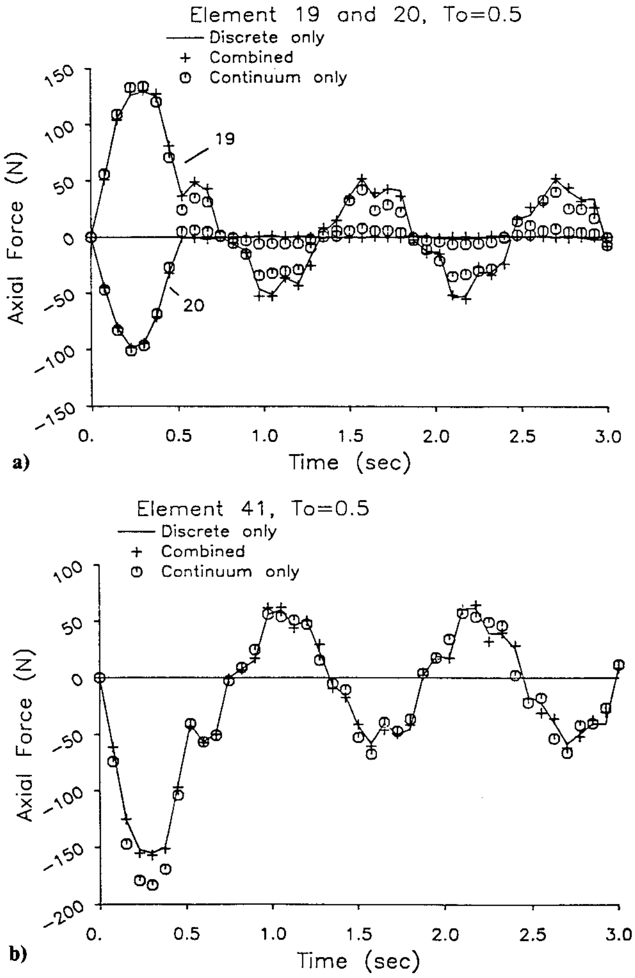


Fig. 8 Axial force in elements of asymmetric cantilever truss beam.

The axial forces produced in the elements of the cell where the load is applied are shown in Fig. 8. The continuum model shows appreciable errors in all the members. On the other hand, the combined model yields excellent results.

C. General Truss Beam Structure

As a final example, a truss structure as shown in Fig. 9 was modeled using conventional discrete finite elements and using the global-local method. This is an example of a structure that is not suitable for using only continuum elements, and is a case where the global-local approach can be most useful.

Table 3 Natural frequencies of general truss beam (radians)

Mode	Discrete only	Global-local three continuum	Global-local three continuum
1	3.49	3.50	3.50
2	22.9	22.7	23.6
3	55.0	54.7	63.2
4	63.0	63.2	63.9
5	87.4	87.1	104.0

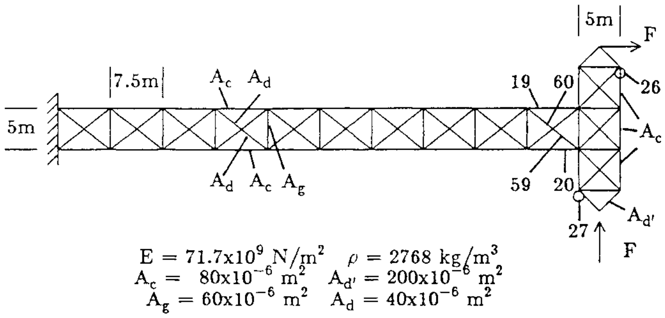


Fig. 9 General truss beam.

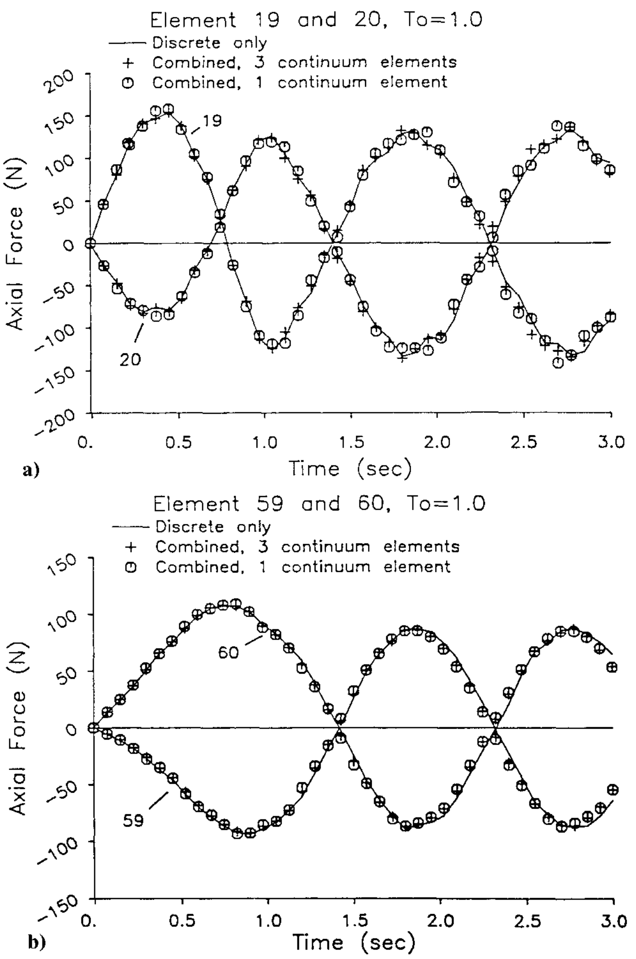


Fig. 10 Axial force in elements of general truss beam.

The nine cells closest to the fixed end of the horizontal section were replaced with continuum elements in the global-local model. The vertical portion of the truss structure was modeled by discrete bar elements. Two global-local models were investigated. In the first, the nine cells were replaced with three continuum elements. In the second, only one continuum element was used to replace all nine cells.

The five lowest natural frequencies were found for the three models. The results are shown in Table 3. It is evident that the use of one continuum beam element to model the whole horizontal truss beam resulted in large errors for the higher modes. With fewer degrees of freedom, the continuum model does not include the higher frequencies of the structure.

A transient analysis was performed on the structure using the same forcing function given by Eq. (17). However, the value of T_0 was changed to 1.0 s for this example. The force was applied as shown in Fig. 9.

The axial forces in the elements marked in Fig. 9 are shown in Fig. 10. As can be seen, the combined models give good results, even for the case when only one continuum element is used.

The use of the continuum element resulted in considerable saving of solution time for this problem. In the discrete model, a total of 56 degrees of freedom are needed to model the system. For the case where nine cells were replaced by one continuum element, only 26 degrees of freedom were needed to describe the structure. This results in great saving in solution time.

VII. Conclusion

A simple method of combining continuum and discrete finite elements together in trusses has been presented and evaluated. The present approach extends the application of continuum models to beam-like structures connected to trusses of arbitrary shapes. Numerical examples show that this approach can save computation time and yield accurate results for the dynamic response of large truss structures.

Acknowledgment

This research was supported by NASA Langley Research Center under Grant NAG-1-581. W. J. Stroud was the technical monitor.

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